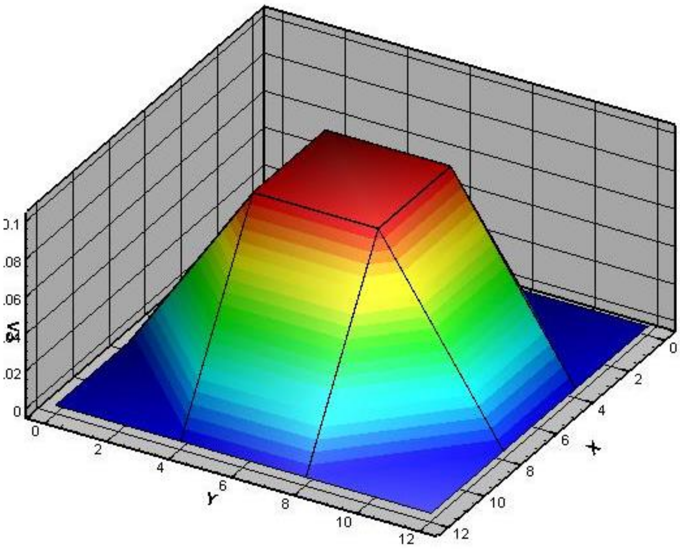
Computational Mechacnis

EN2340 Final Project

<Computation of Membrane deflection>



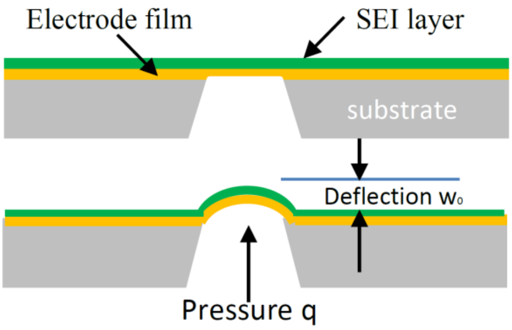
Insun Yoon

1.Project Objective

Implement pressurized membrane deflection behavior into FEACHEAP.

2. Background

Membrane deflection measurement is a validated method to measure membrane's stiffness and residual stress. Personally, I'm using this method to measure mechanical property of Solid Electrolyte Interphase.



The background mechanics on membrane deflection is well described on S. Timoshenko's book "Theory of Plates and Shells", 1964, and skipped in this report.

3. Challenge

In membrane geometry, the thickness of membrane is thin enough compared to other dimensions. Therefore in this calculation, the model should have 2-dimensional geometry with uniform thickness. The degrees of freedom however, should be 3-dimensional including vertical deflection.

The delfction of membrand has non-linear behavior. Newton-Rhapson method is needed to calculate displacement. In case of large deflection, careful settings of material properties, dimensions and time increment is essential.

If the membrane does not have any residual stress or pre-strain, the stiffness to vertical direction has very small value. Since the model should use Newton-Rhapson interation, this causes very unstable calculation at the very first step.

The membrane is constrained all degrees of freedom at all edge. Needless to say, the membrane should contain fine enough mesh to calculate the maximum deflection.

4. Equilibrium Equations and Finite Element Equations

Derivation is writtien on the next page.

! Derivation is hand-written and submitted. Images taken from my scratch papers and might have typos. From the top, it shows discretized potential energy, each residual of 3 DOFs, and stiffness matrix.

$\int P w \, dx\, dy$

discretize $u = N^a u^a \quad v = N^a v^a \quad w = N^a w^a$

$$\pi(u,v,w) = \int \frac{Et}{2(1+\nu)(1-\nu)} \cdot \Big[N_x^a {u^a}^2 + N_x^a u^a (N_x^a w^a)^2$$
$$+ \frac{1}{4}(N_x^a w^a)^4 + 2\nu (N_x^a u^a)(N_y^a v^a) + \nu (N_x^a u^a)(N_y^a w^a)^2$$
$$+ \nu (N_y^a v^a)(N_x^a w^a)^2 + (N_y^a v^a)^2 + (N_y^a v^a)(N_y^a w^a)^2$$
$$+ \frac{1}{4}(N_y^a w^a)^4 + \frac{1}{2}(1-\nu)(N_y^a u^a)^2 + \frac{1}{2}(1-\nu)(N_x^a v^a)^2$$
$$+ \frac{1}{2}(N_x^a w^a)^2 (N_y^a w^a)^2 + (1-\nu)(N_y^a u^a)(N_x^a v^a)$$
$$+ (1-\nu)(N_x^a v^a)(N_x^a w^a)(N_y^a w^a)$$
$$+ (1-\nu)(N_y^a u^a)(N_x^a w^a)(N_y^a w^a) \Big] dx\,dy$$
$$- \int P N^a w^a \, dx\,dy$$

$$R_u^a = \int \smile \Big[2(N_x^k u^k) N_x^a + 2\nu N_y^k v^k N_x^a + (1-\nu)(N_y^k u^k) N_y^a + (1-\nu)(N_x^k v^k) N_y^a + (N_x^k w^k)^2 N_x^a + \nu (N_y^k w^k)^2 N_x^a + (1-\nu) N_y^a \Big] dx\,dy$$

$$R_v^a = \int \smile \Big[2(N_y^k v^k) N_y^a + 2\nu N_x^k u^k N_y^a + (1-\nu)(N_x^k v^k) N_x^a + (1-\nu)(N_y^k u^k) N_y^a + \nu (N_x^k w^k)^2 N_y^a + (N_y^k w^k)^2 N_y^a + (1-\nu) N_x^a \Big] dx\,dy$$

$$R_w^a = \int \smile \Big[(N_x^k w^k)^3 N_x^a + (N_y^k w^k)^3 N_y^a + (N_y^k w^k)^2 (N_x^l w^l) N_x^a + (N_x^k w^k)^2 (N_y^l w^l) N_y^a$$
$$+ (N_x^k u^k + \nu N_y^k v^k)(N_x^l w^l) N_x^a + (\nu N_x^k u^k + N_y^k v^k)(N_y^l w^l) N_y^a$$
$$+ (N_x^k w^k) N_y^a + (N_y^k w^k) N_x^a \Big] dx\,dy - \int p N^a \, dx\,dy$$

now ↓

$\smile \Rightarrow \frac{Et}{2(1-\nu^2)}$

$\int a \mid b \quad u \quad u$

$K_{aubv} \quad K_{avbv} \quad K_{awbv}$

⇓ $\frac{\partial R_u}{\partial v} \quad \frac{\partial R_v}{\partial v} \quad \frac{\partial R_w}{\partial v}$

$$K_{aubv} = \frac{\partial R_u^a}{\partial v^b} = \int \smile \big[2\nu N_y^b N_x^a + (1-\nu) N_y^b N_y^a \big]$$

$$K_{avbv} = \frac{\partial R_v^a}{\partial v^b} = \int \smile \big[2 N_y^b N_y^a + (1-\nu) N_x^b N_x^a \big]$$

$$K_{awbv} = \int \smile \big[\nu N_y^b N_x^a (N_x^k w^k) + N_y^b N_y^a (N_y^k w^k) \big]$$

~~$K_{aubw} = \frac{\partial R_u^a}{\partial w^b} \int (N_x^k w^k) N_x^a \ldots +$~~

$$K_{aubw} = \frac{\partial R_u^a}{\partial w^b} = \int \smile \big[(N_x^k w^k) N_x^b N_x^a + \nu (N_y^k w^k) N_y^b N_x^a \big] dx\,dy$$

$$K_{avbw} = \frac{\partial R_v^a}{\partial w^b} = \int \smile \big[\nu (N_x^k w^k) N_x^b N_y^a + (N_y^k w^k) N_y^b N_y^a \big]$$

$$K_{awbw} = \frac{\partial R_w^a}{\partial w^b} = \int \smile \Big[(N_x^k w^k)^2 N_x^b N_x^a + (N_y^k w^k)^2 N_y^b N_y^a$$
$$+ 2(N_y^k w^k)(N_x^l w^l) N_y^b N_x^a$$
$$+ (N_y^k w^k)^2 N_x^b N_x^a$$
$$+ 2(N_x^k w^k)(N_y^l w^l) N_x^b N_y^a$$
$$+ (N_x^k w^k)^2 N_y^b N_y^a$$
$$+ (N_x^k u^k + \nu N_y^k v^k) N_x^b N_x^a$$
$$+ (\nu N_x^k u^k + N_y^k v^k) N_y^b N_y^a + N_x^b N_y^a + N_y^b N_x^a$$

$K_{aibk} = \frac{R_i^a}{dw_k^b}$ 2D N-R

$K_{aibu} \, du^b = R_i^a$

$$K_{aubu} = \frac{\partial R_u^a}{\partial u^b} = \int \smile \big[2 N_x^b N_x^a + (1-\nu)(N_y^b N_y^a) \big] dx\,dy$$

$$K_{avbu} = \frac{\partial R_v^a}{\partial u^b} = \int \smile \big[2\nu N_x^b N_y^a + (1-\nu) N_y^b N_x^a \big] dx\,dy$$

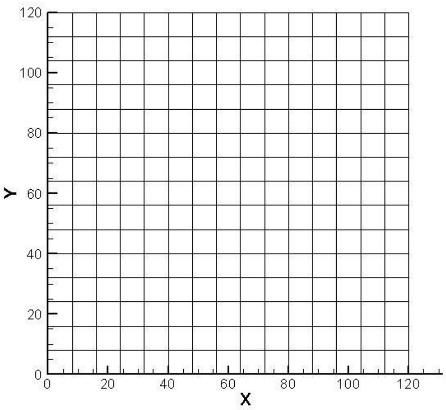
$$K_{awbu} = \frac{\partial R_w^a}{\partial u^b} = \int \smile \big[(N_x^b N_x^a)(N_x^l w^l) + \nu N_x^b N_y^a (N_y^l w^l) \big] dx\,dy$$

↳ for du

5. Calculation and Validation

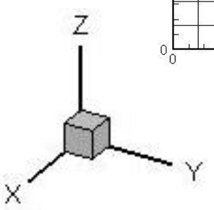
FEA on membrane with 100GPa of Young's modulus, 0.3 of Poisson's ratio, and under 137880 Pa (20PSI) of pressure is executed. The membrand has geometry of 120um square and 20nm thickness. Total time step was 100 and pressure was applied gradually with time.

Small amount of strain is applied to detour no-stiffness problem to veritcal direction.



Initial 2-D mesh

120um X 120um square devided by 15 X 15



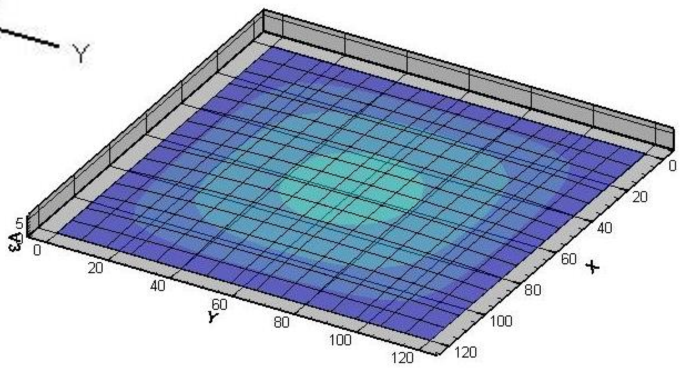


Fig. 1 Membrane deflected at time step 20/100 Maximum deflection 1.6um

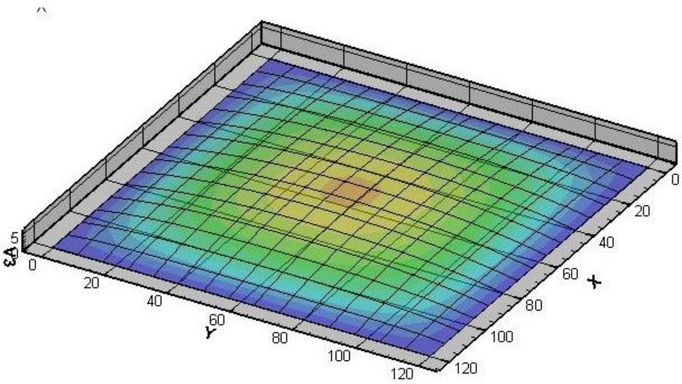


Fig. 2 Membrane deflected at time step 60/100 Maximum deflection 5.07um

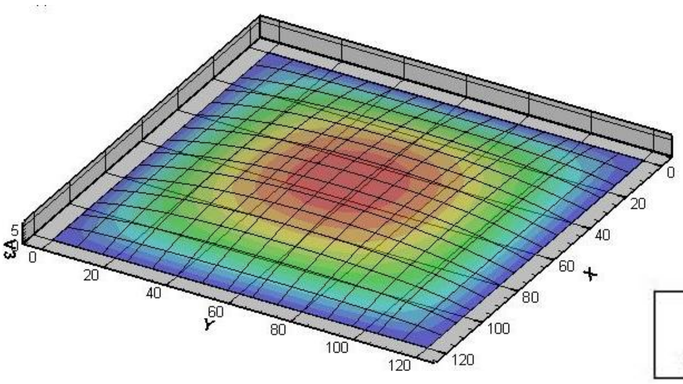
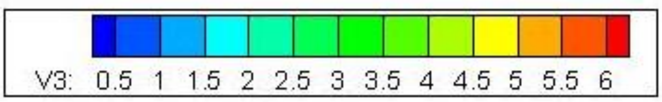


Fig. 3 Membrane deflected at time step 100/100 Maximum deflection 6.41um

!All dimensions in um. V3:Vertical deflection



Maximum deflection at each step is extracted to compare analytical solution.

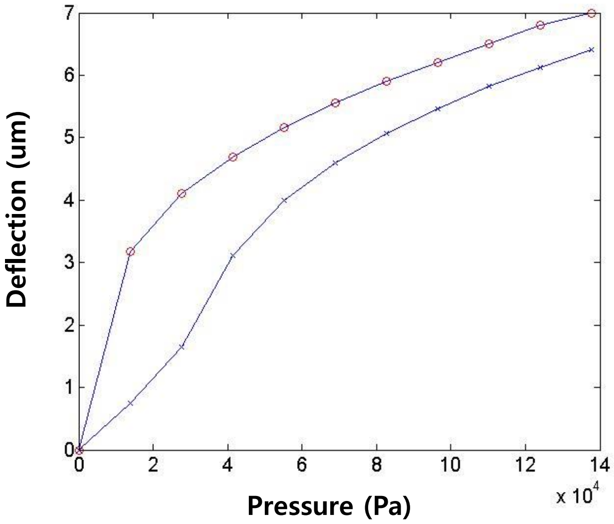


Fig. 4 Comparison of analytical solution and computation.

Figure 4 shows analytical and computational solutions. The two values show significant difference at first 30 steps. After 40 steps, the two values have a certain difference. One reasoning of these differences can arise from small strain applied in FEA. Especially when the applied pressure is small, strain (or residual stress) plays a big roll. As pressure increases, the difference becomes stable and we can guess this is coming from pre-strain.

6. Further development

First, this FEA should be tested with various dimensions and pressure to assure this analysis is valid. I could not finish this task due to time limitation.

Second, the FEA code can be improved by implementing the factors of residual stress, pre-strain. In reality, every membrane is pre-strained or has residual stress. Also the case of pre-deflected model can be added.

Third, I should find more clever way to detour the small stiffness to vertical direction problem.